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NOTES ON THE CALCULATION OF THE OSMOTIC PRESSURE OF EXPRESSED VEGETABLE SAPS FROM THE DEPRESSION OF THE FREEZING POINT, WITH A TABLE FOR THE VALUES OF P FOR $\Delta = 0.001^{\circ}$ TO $\Delta = 2.999^{\circ 1}$

J. ARTHUR HARRIS AND ROSS AIKEN GORTNER

By far the simplest method of determining the osmotic pressure of expressed vegetable saps is that of the depression of the freezing point by use of the Beckmann² apparatus or some substitute procedure.³ Particular emphasis has been laid upon the practicability and accuracy of the freezing point method by Lewis.⁴ Renner⁵ has done a good service to biologists by pointing out that inconsistencies between the plasmolytic and the cryoscopic method are, in considerable part at least, due to the difference between *weight normal* and *volume normal* solutions. Thus one objection to the cryoscopic method is removed. In this note we desire merely to call attention to an important factor which has been very generally neglected by biologists in the calculations of the osmotic pressure from the experimentally determined data, and to lighten somewhat the routine of those working in this field by the publication of a little table from which P for $\Delta = 0.001^{\circ}$ to $\Delta = 2.999^{\circ}$ C. may be at once determined.

The freezing point of a solution is the temperature at which ice and solution exist in equilibrium. Pure water separates upon freezing, hence the liquid which remains, and whose temperature in equilibrium with the mass of ice crystals is read from the thermometer, is not the

¹ From the Station for Experimental Evolution, The Carnegie Institution of Washington.

² See any textbook of physical chemistry for the general methods. For some special points of technique in dealing with vegetable saps see Gortner and Harris, Notes on the Technique of the Determination of the Depression of the Freezing Point of Vegetable Saps, *Plant World*, **17**: 49-53. 1914.

³ Dixon, H. H. and Atkins, W. R. G. *Sci. Proc. Roy. Dublin Soc.*, n. s. **12**: 275-311, loc. cit. **13**: 49-62. 1911.

⁴ Lewis, G. N. The Osmotic Pressure of Concentrated Solutions, and the Laws of the Perfect Solution. *Journ. Amer. Chem. Soc.* **30**: 668-683. 1908.

⁵ Renner, G. Über die Berechnung des osmotischen Druckes. *Biolog. Centralbl.* **32**: 486-504. 1912. See also review of C. A. Shull, *Bot. Gaz.* **56**: 444. 1913.

same but is more concentrated than the original solution. In consequence the observed depression is lower than the true freezing point of the solution under investigation. That this factor may become of considerable importance in physiological work is shown at once by a comparison of corrected and uncorrected values taken from unpublished observations:

	Δ' Depression of the freezing point as read from the thermometer	Δ Depression corrected for weight of ice separated	Difference between corrected and uncorrected depressions.
Apple.....	1.209°	1.166°	0.043°
Apple.....	1.131°	1.093°	0.038°
Pear.....	1.332°	1.289°	0.043°
Pear.....	1.461°	1.412°	0.049°
Passiflora.....	0.738°	0.710°	0.028°
Passiflora:.....	0.630°	0.607°	0.023°

Such differences cannot be neglected. They may be minimized by "salting" with a few crystals of frost after the solution has been cooled to below its freezing point, or an absolute correction may be made where the volume of the *solvent*, not the volume of the *solution*, and the amount of under-cooling (*i. e.*, the difference between the point to which the mercury fell before the separation of ice began, and the maximum temperature reached in the system, solution + ice) are known. For 1 cc. of solution,⁶ volume v , having a density of d , freezing at Δ' after an under-cooling of u , and yielding total solids s , the weight of water is, of course $d - s$. Since for each degree of under-cooling 12.5 g. of water separates as ice in each liter, 1/80th of the weight of the solvent is removed and the concentration of the solution is proportionally increased. Hence the correction

$$\frac{\left(v - \frac{uv}{80}\right) \Delta'}{v} = \Delta$$

or in terms of freezing point lowering and undercooling only

$$\Delta = \Delta' - 0.0125 u \Delta'$$

must be applied.

Some slight differences are to be noted in the literature in the method of computing the osmotic pressure in atmospheres (P) from

⁶ For simplicity of calculation merely unit-volume is adopted. In practice several cubic centimeters must be used to secure accurate figures for Δ' and s .

Δ —inconsistencies which for the most part result in variations of P less than those attributable to experimental error. In computing the accompanying table we have employed the formula of Lewis⁷ quoted by Morgan.⁸ To quote from Lewis⁹

"The osmotic pressure may be obtained at once from the freezing point by means of the equation

$$\pi = 12.06\Delta - 0.021\Delta^2,$$

where π is the osmotic pressure in atmospheres and Δ is the lowering of the freezing point in centigrade degrees. From this equation the osmotic pressure of any solution up to 10 or 15 times normal may be obtained with an accuracy which depends only upon the precision of the freezing point determinations and upon the accuracy of the value used for the heat of fusion of ice. Since the error in the latter quantity is probably not more than 0.1 per cent, it is obvious that except for the most dilute solutions osmotic pressures may be found in this way with an accuracy which is more than ten times as great as Morse and Frazer claim for their direct measurements."

The osmotic pressures corresponding to depressions of hundredths of degrees may be read directly from the table. Furthermore *for all practical purposes* the first differences $\times 0.1$ may be taken as 0.012, hence pressures when Δ is read to thousandths of a degree may be at once determined. Thus suppose Δ to be 1.244° . For $\Delta = 1.24^\circ$ $P = 14.92^\circ$. $14.92 + (4 \times 0.012) = 14.968$. The exact value by direct computation is 14.970. Or again, $\Delta = 1.248^\circ$; $P = 14.92 + (8 \times 0.012) = 15.016$ as compared with the exact value 15.018.¹⁰

⁷ Lewis, G. N. Loc. cit.

⁸ Morgan, J. L. R. "Elements of Physical Chemistry," 4th ed., 1908, p. 180.

⁹ Lewis, G. N. Loc. cit., pp. 670-671.

¹⁰ Since the biologist's error in the determination of Δ may be at least 0.001°C . which affects P in the second decimal place, it is idle to write down the pressures to more than four significant figures. The entries in the tables were, of course, calculated to a sufficient number of places to avoid the possibility of arithmetical error. In copying off the table the final figure has been modified where needful.

TABLE OF OSMOTIC PRESSURES IN ATMOSPHERES FOR DEPRESSION OF THE FREEZING POINT TO 2.999° C.

Δ	Hundredths of degrees, centigrade									
	0		2	3	4	5	6	7	8	9
0.0	0.000	0.121	0.241	0.362	0.482	0.603	0.724	0.844	0.965	1.085
0.1	1.206	1.327	1.447	1.568	1.688	1.809	1.930	2.050	2.171	2.291
0.2	2.412	2.532	2.652	2.772	2.893	3.014	3.134	3.255	3.375	3.496
0.3	3.616	3.737	3.857	3.978	4.098	4.219	4.339	4.459	4.580	4.700
0.4	4.821	4.941	5.062	5.182	5.302	5.423	5.543	5.664	5.784	5.904
0.5	6.025	6.145	6.266	6.386	6.506	6.627	6.747	6.867	6.988	7.108
0.6	7.229	7.349	7.469	7.590	7.710	7.830	7.951	8.071	8.191	8.312
0.7	8.432	8.552	8.672	8.793	8.913	9.033	9.154	9.274	9.394	9.514
0.8	9.635	9.755	9.875	9.995	10.12	10.24	10.36	10.48	10.60	10.72
0.9	10.84	10.96	11.08	11.20	11.32	11.44	11.56	11.68	11.80	11.92
1.0	12.04	12.16	12.28	12.40	12.52	12.64	12.76	12.88	13.00	13.12
1.1	13.24	13.36	13.48	13.60	13.72	13.84	13.96	14.08	14.20	14.32
1.2	14.44	14.56	14.68	14.80	14.92	15.04	15.16	15.28	15.40	15.52
1.3	15.64	15.76	15.88	16.00	16.12	16.24	16.36	16.48	16.60	16.72
1.4	16.84	16.96	17.08	17.20	17.32	17.44	17.56	17.68	17.80	17.92
1.5	18.04	18.16	18.28	18.40	18.52	18.64	18.76	18.88	19.00	19.12
1.6	19.24	19.36	19.48	19.60	19.72	19.84	19.96	20.08	20.20	20.32
1.7	20.44	20.56	20.68	20.80	20.92	21.04	21.16	21.28	21.40	21.52
1.8	21.64	21.76	21.88	22.00	22.12	22.24	22.36	22.48	22.60	22.72
1.9	22.84	22.96	23.08	23.20	23.32	23.44	23.56	23.68	23.80	23.92
2.0	24.04	24.16	24.28	24.40	24.52	24.64	24.76	24.88	24.99	25.11
2.1	25.23	25.35	25.47	25.59	25.71	25.83	25.95	26.07	26.19	26.31
2.2	26.43	26.55	26.67	26.79	26.91	27.03	27.15	27.27	27.39	27.51
2.3	27.63	27.75	27.87	27.99	28.11	28.23	28.34	28.46	28.58	28.70
2.4	28.82	28.94	29.06	29.18	29.30	29.42	29.54	29.66	29.78	29.90
2.5	30.02	30.14	30.26	30.38	30.50	30.62	30.74	30.86	30.98	31.09
2.6	31.21	31.33	31.45	31.57	31.69	31.81	31.93	32.05	32.17	32.29
2.7	32.41	32.53	32.65	32.77	32.89	33.00	33.13	33.25	33.36	33.48
2.8	33.60	33.72	33.84	33.96	34.08	34.20	34.31	34.43	34.56	34.68
2.9	34.79	34.91	35.04	35.16	35.27	35.39	35.51	35.63	35.75	35.87